Utilisation des graphes pour la caractérisation topologique et hydrologique des réseaux de fractures

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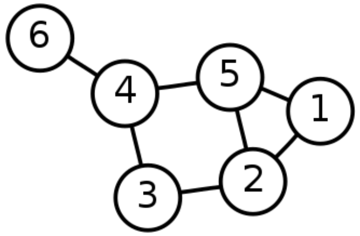
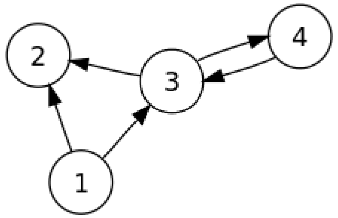
Graphs Theory

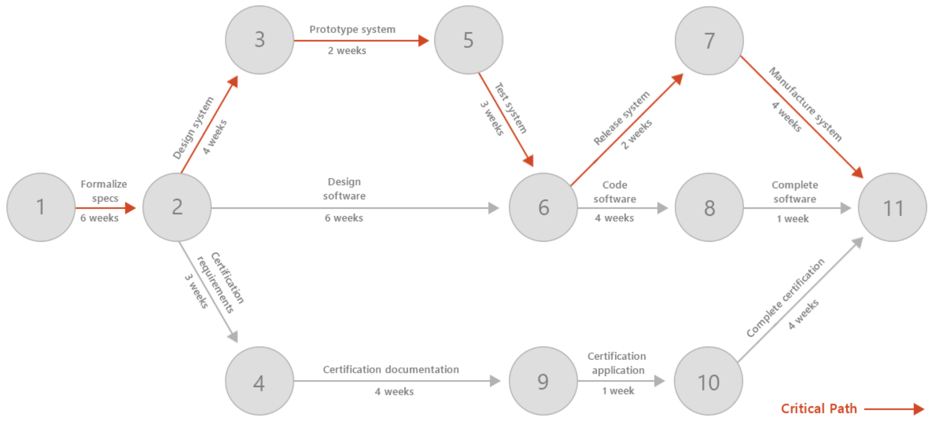
⇒ Mathematical model where objects (**Vertex**) communicate together (**Edges**), forming a network : $G = (V, E)$





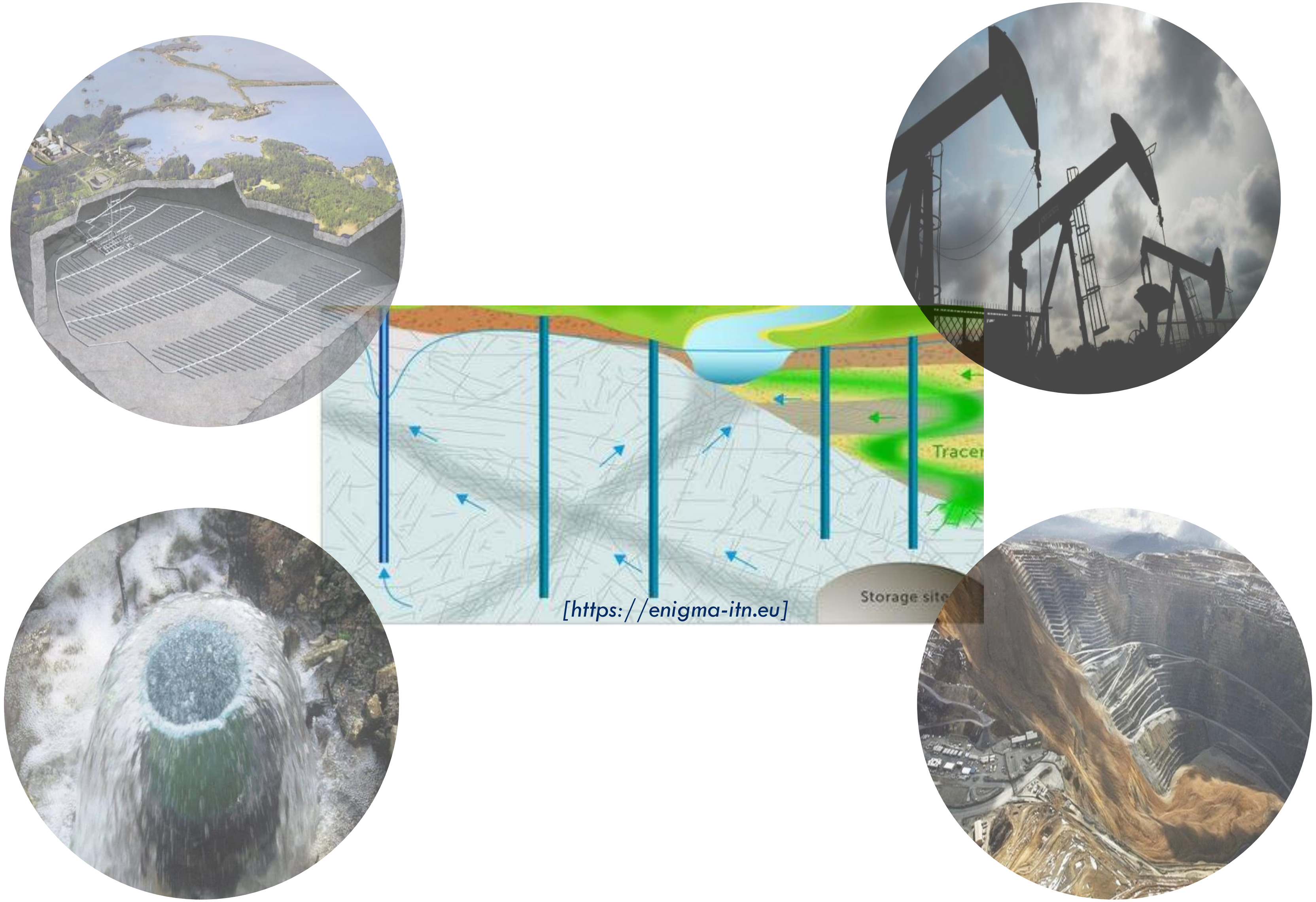
⇒ Used everywhere

[http://www.bretagne.bzh]



[https://www.officetimeline.com]

Fracture Network



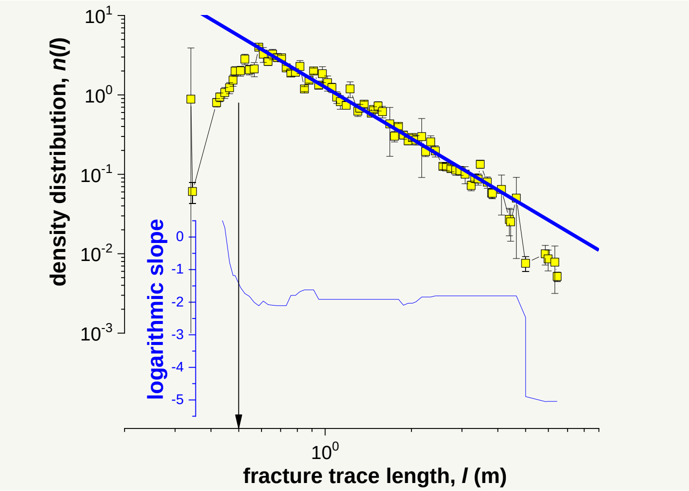
[https://enigma-itn.eu]

Discrete Fracture Network Framework

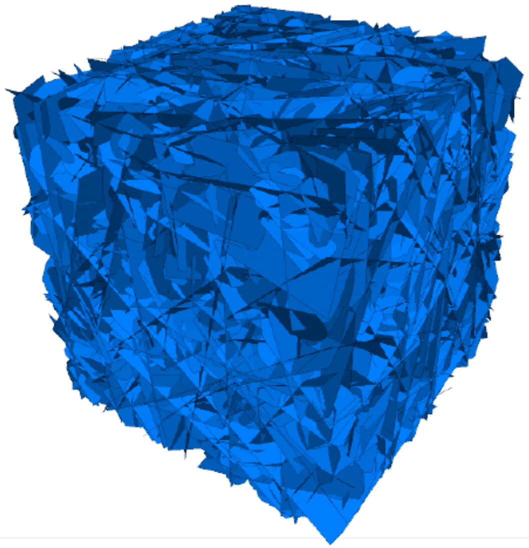
Data acquisition

[provided by SKB]

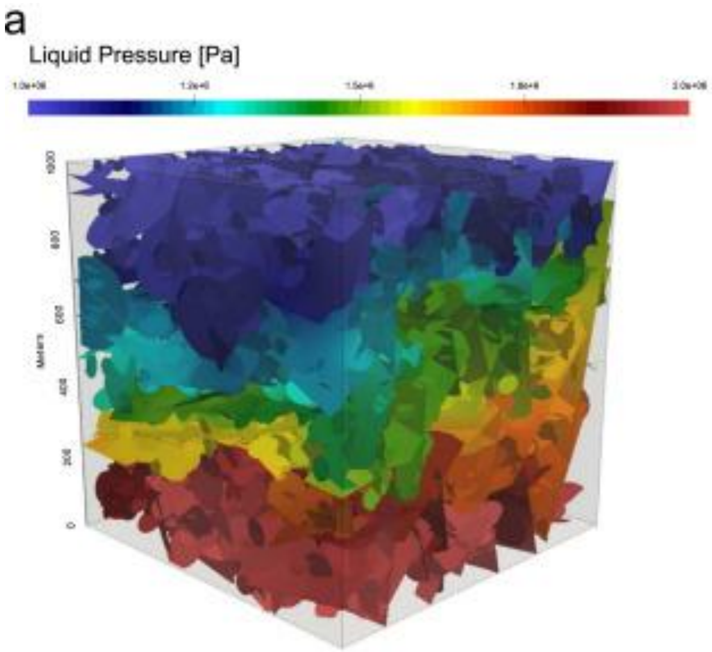
+ statistical analysis



Geometric model (DFN)
+ associated properties



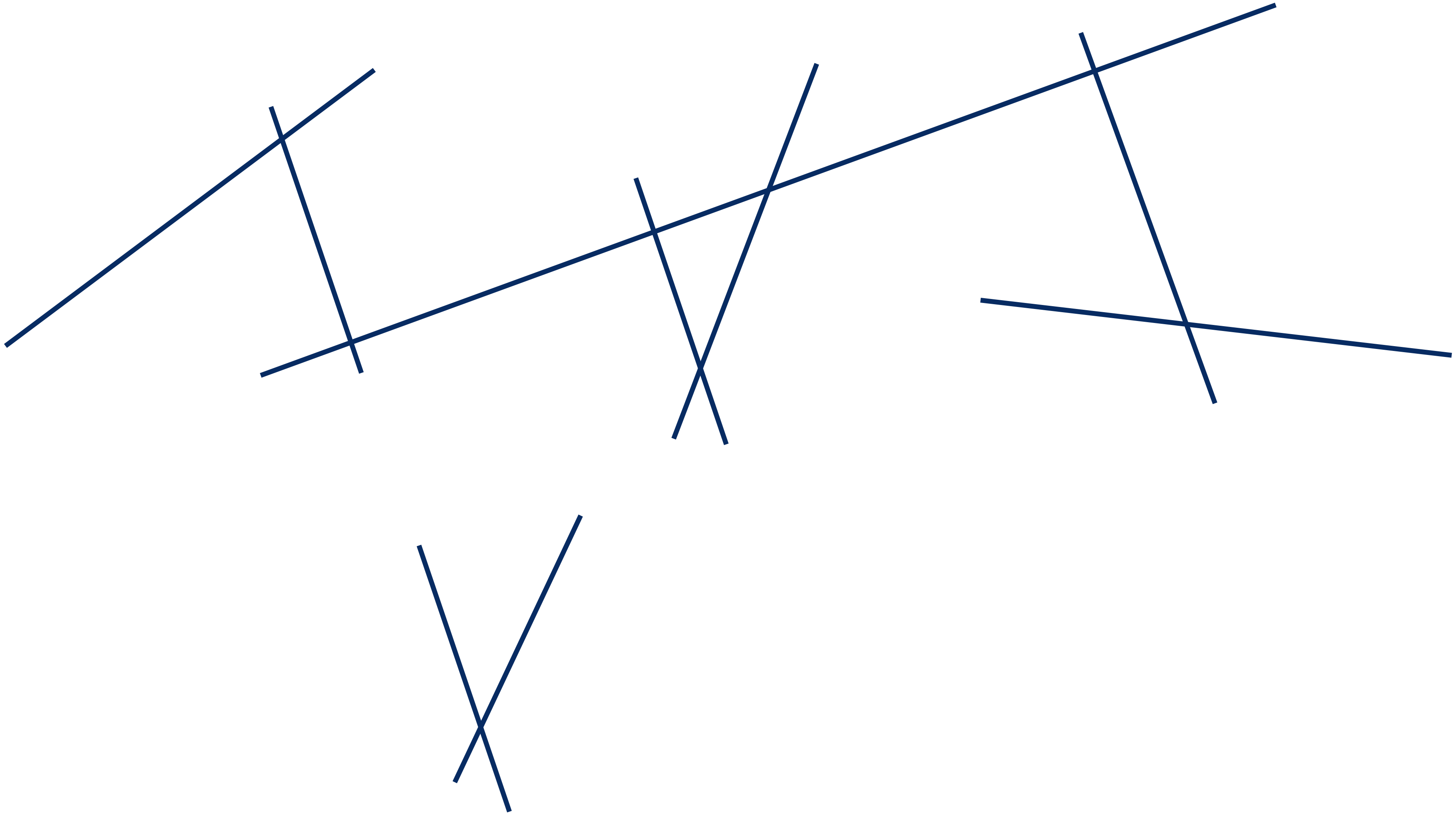
Predictions



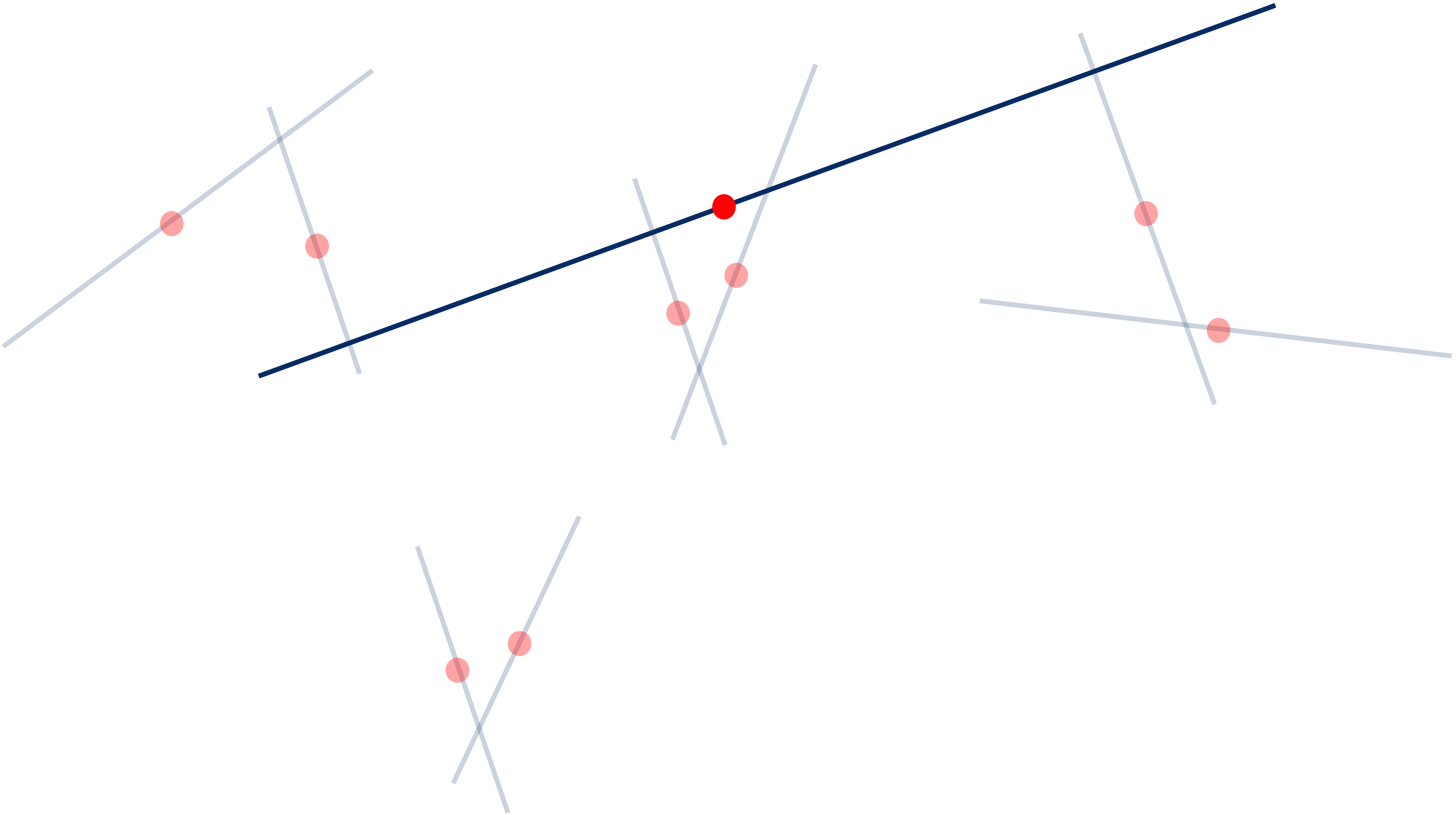
[dfnWorks]

⇒ Computationally expensive
⇒ Need for several realisations

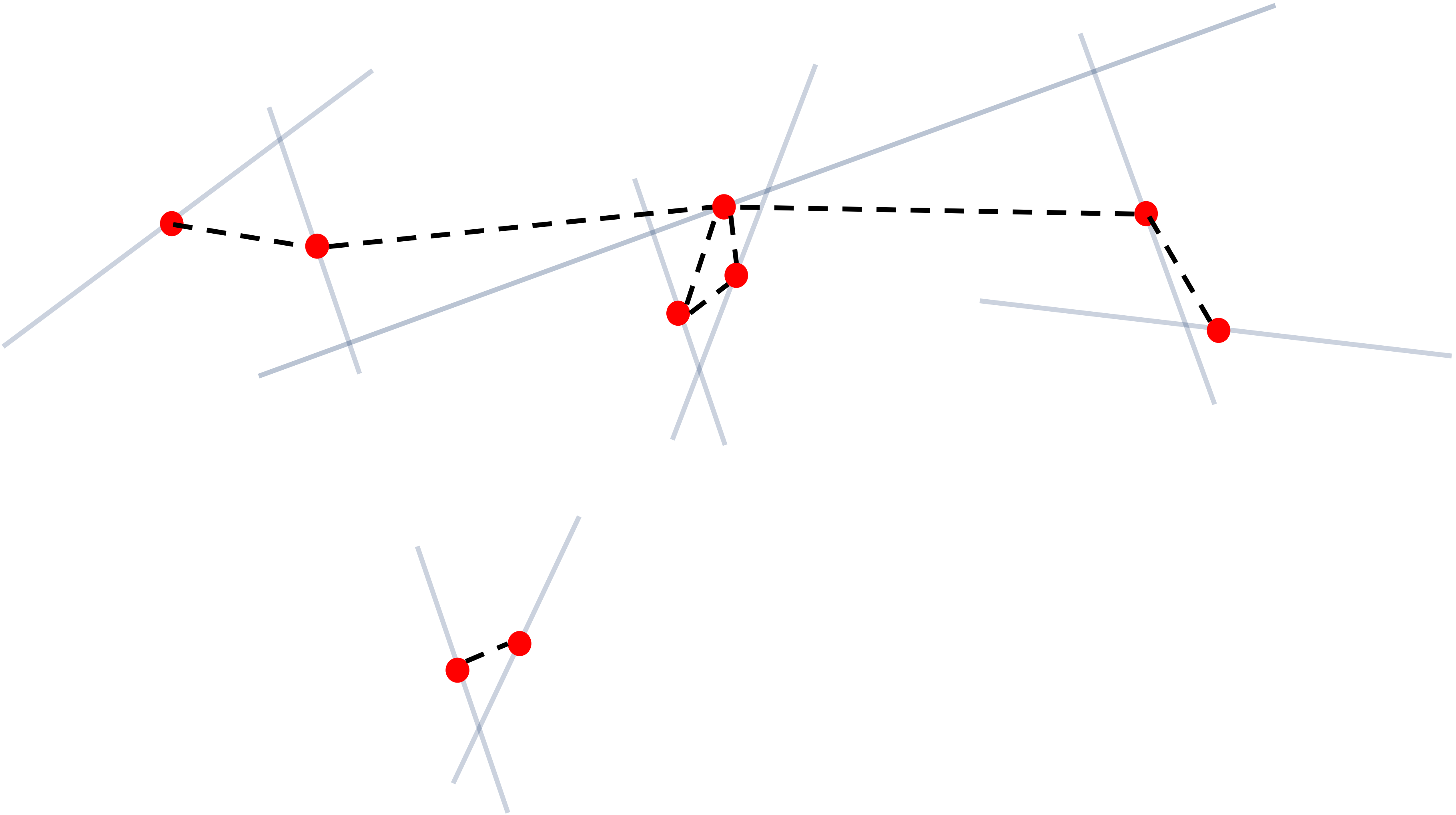
Discrete Fracture Network



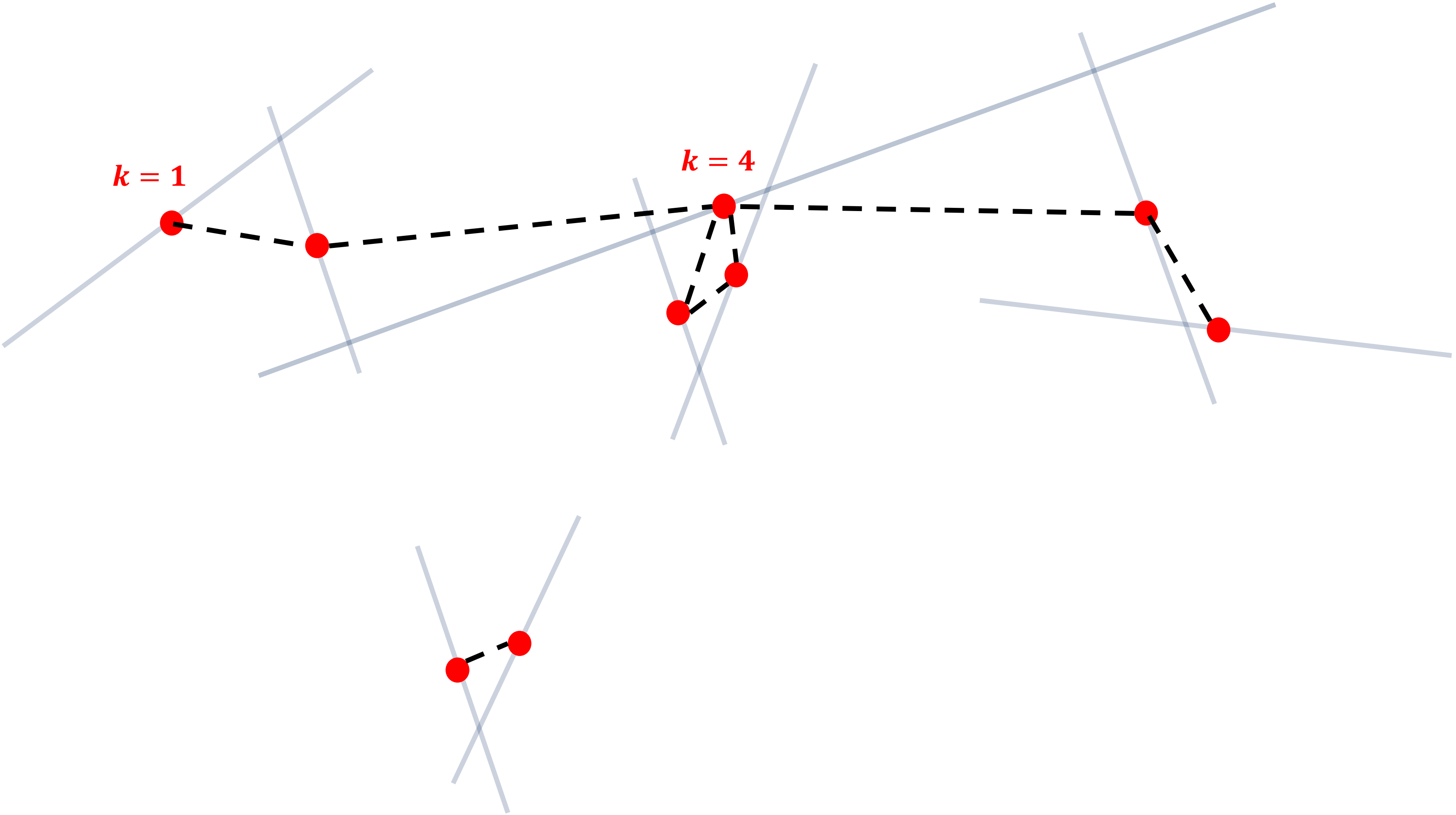
Graph Fracture Network



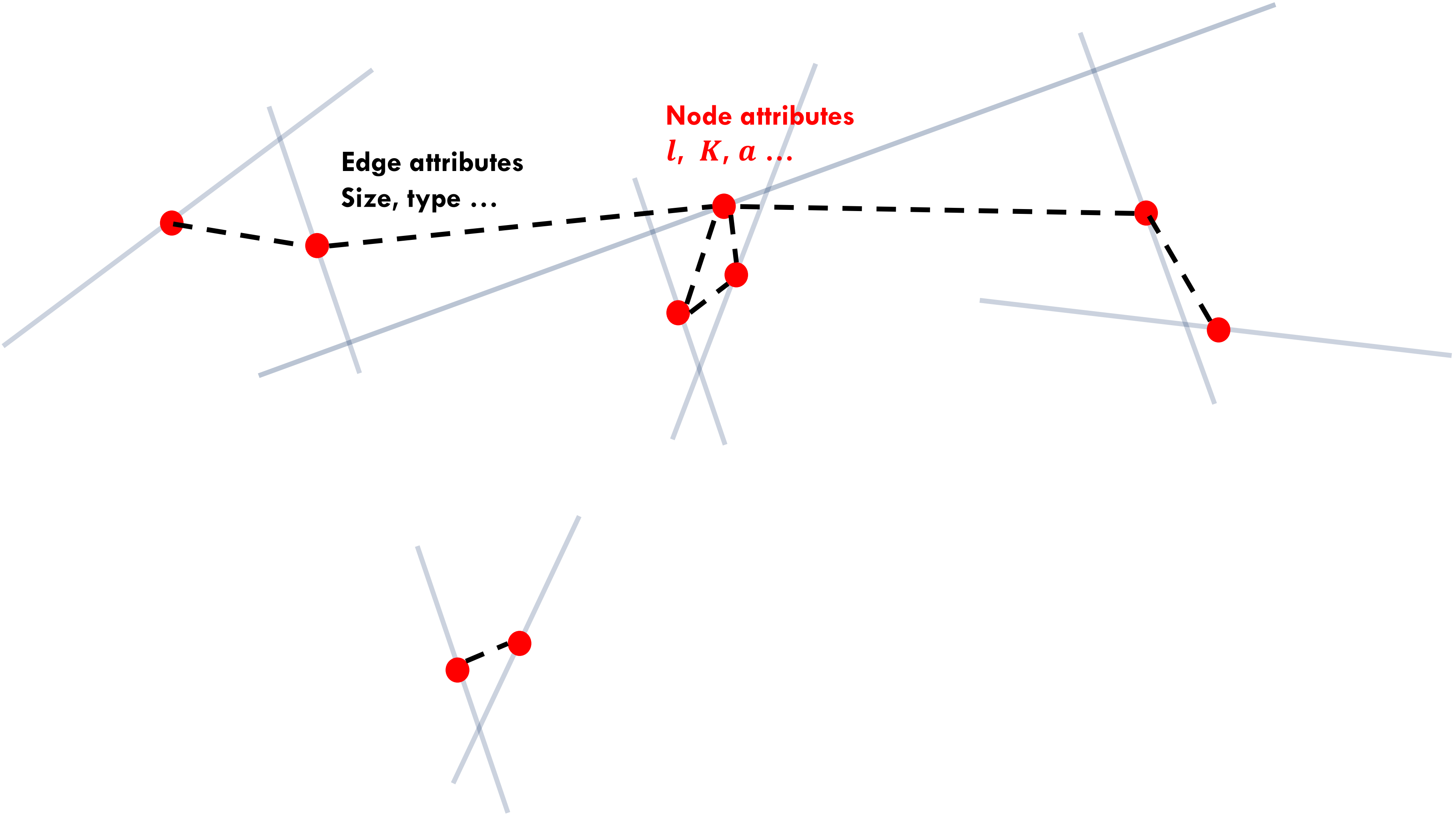
Graph Fracture Network



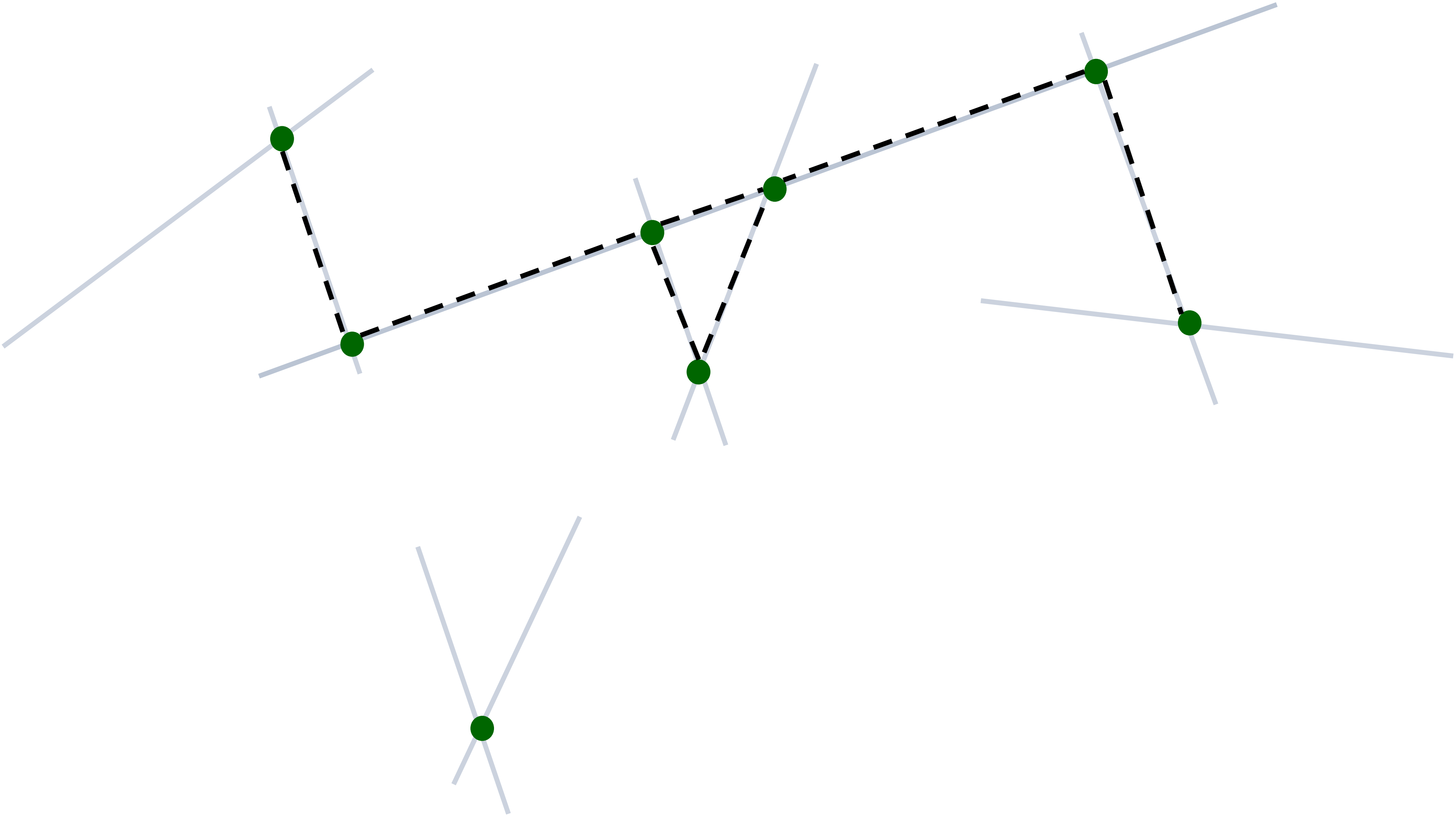
Graph Fracture Network



Graph Fracture Network

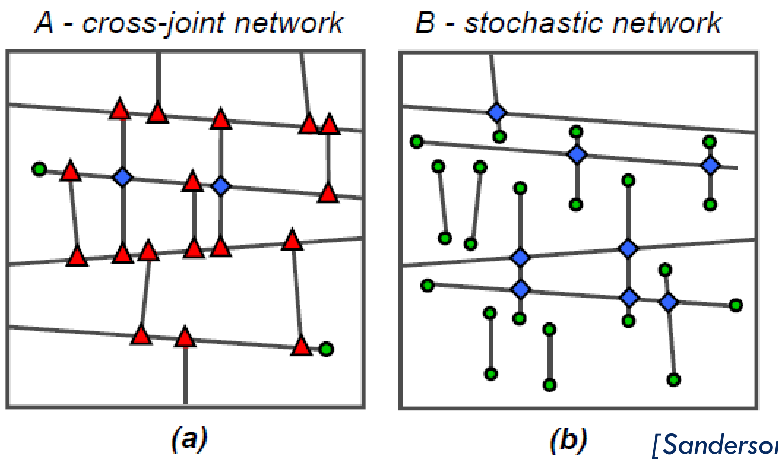


Graph Fracture Network



Topology

⇒ Two systems can contain the same geometrical elements (orientation, size…), with different topologies



[Sanderson, Nixon, 2015]

⇒ Topology is related to connectivity and can have important mechanical and hydrological consequences

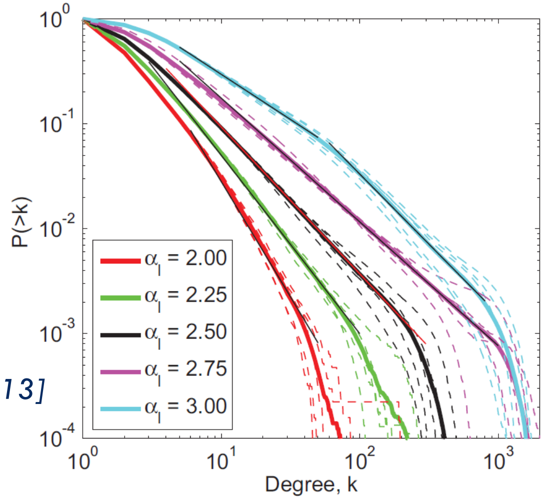
⇒ Quantified by different indicators

⇒ Compare those indicators for data and model, to validate the model

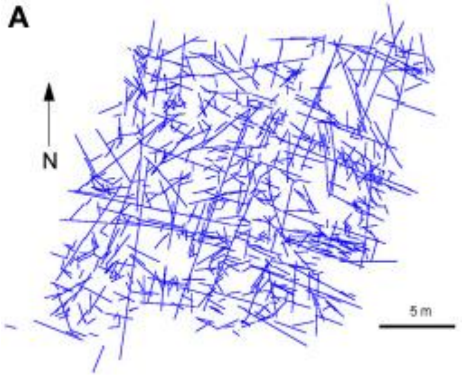
Topology – Compare Data to Models

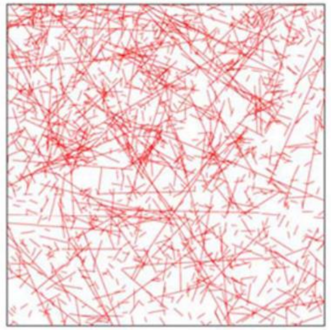
⇒ **Cumulative degree distribution**

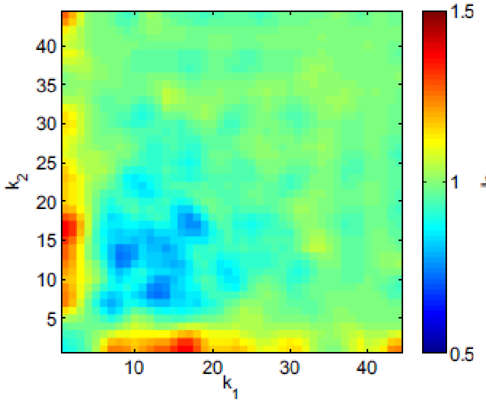
Fraction of nodes with degree higher than k

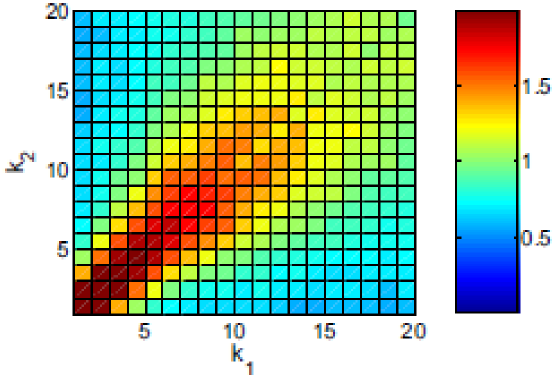


[Andresen et al., 2013]









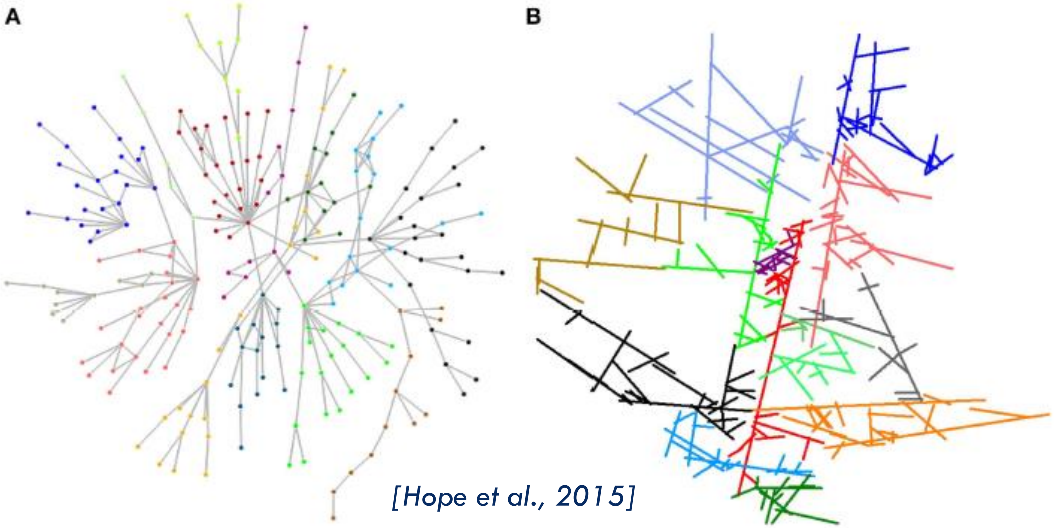
⇒ **Degree correlations**

$$C(k_1, k_2) = \frac{P(k_1, k_2)}{P_R(k_1, k_2)}$$

Probability for a fracture of degree k_1 to be linked with a fracture of degree k_2

[Andresen et al., 2013]

Topology – Compare Data to Models



[Hope et al., 2015]

⇒ **Community structure**

Identify parts of the network which are highly interconnected but have relatively few connections to other parts of the network

⇒ **Efficiency**

$$E = \frac{1}{N(N-1)} \sum_{(i,j)\in N, i\neq j} \frac{1}{d_{ij}}$$

Measures how well the different parts of the network are connected

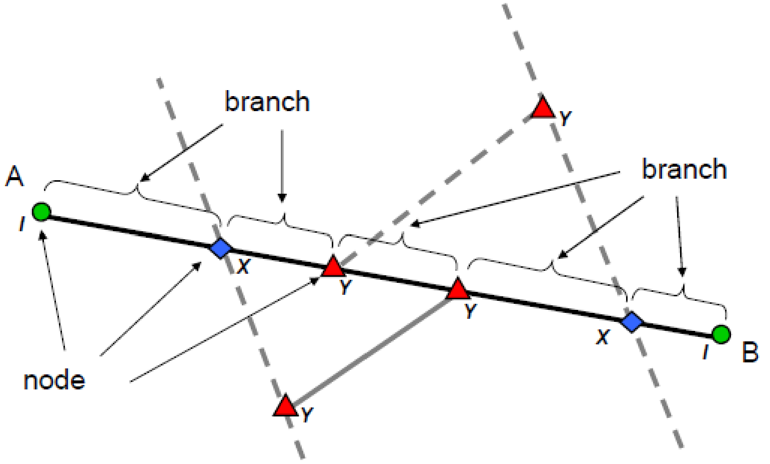
⇒ **Clustering**

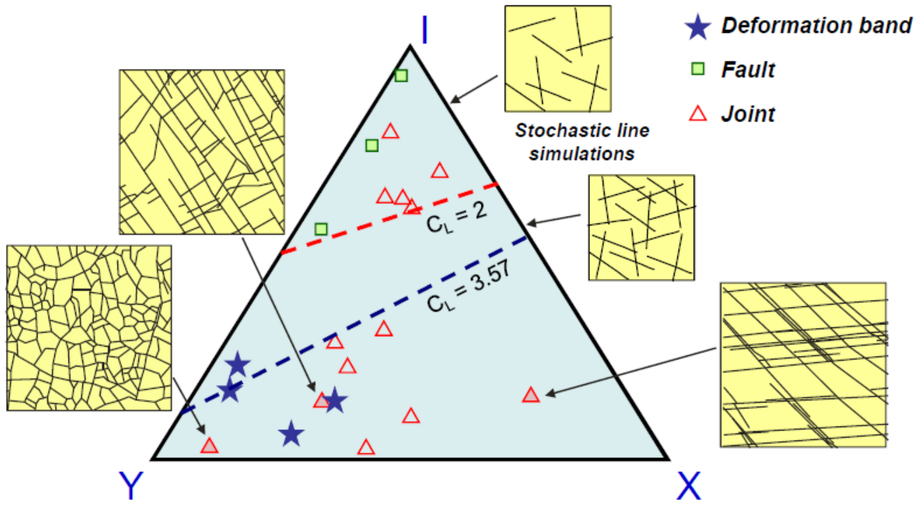
$$C = \frac{1}{N} \sum_{i=1}^{N} \frac{2E_{NN,i}}{k_i(k_i-1)}$$

Quantify how well the network is connected on a neighbor-to neighbor scale

Topology – Compare Data to Models

⇒ **Connectivity** : $C_L = \frac{2(N_Y+N_X)}{N_L}$ (number of connections per line)





[Sanderson, Nixon, 2015]

Percolation threshold :

- $C_L < 2$: no possible spanning cluster
- Random line of fixed length : $C_L = 3.57$ *[Balberg et al, 1984]*

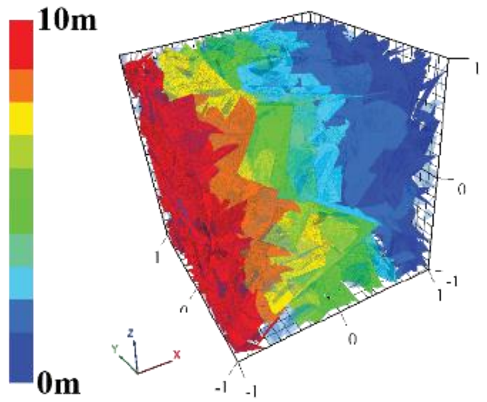
Assessing DFN hydrological properties with Graphs

- Numerical flow and transport simulations in DFN (ex : with H2OLAB).

Hydrological caracterisation of the fracture network :

- Equivalent permeability K_{eq}
- Flow repartition (channeling indicator)
- Breakthrough curves

$\Rightarrow$ Computational cost for meshing, flow and transport computation.
$\Rightarrow$ Several realisations of one statistical model.



- Fracture network is channeled, some structures are more important than others (network backbone).

How can we simplify the fracture network using graphs ?

Assessing DFN hydrological properties with Graphs

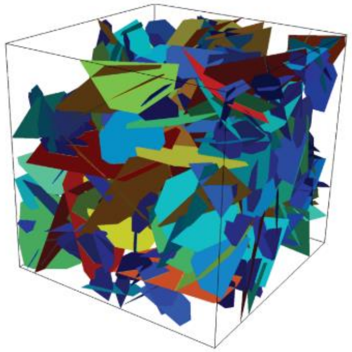
⇒ **Tool for network reduction**

Identification of important structures (topological shortest paths, physical shortest paths)

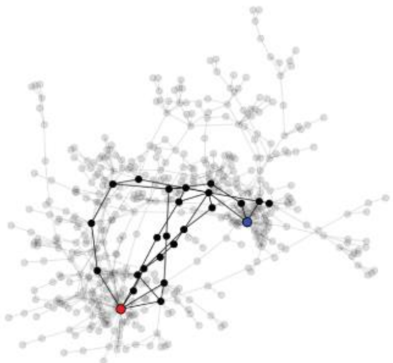
⇒ First arrival times

Pros : Reduced computational times

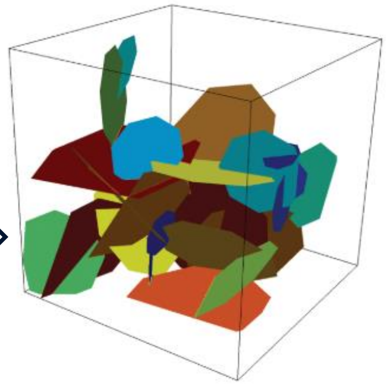
Cons : Loss of information (late arrival times)



Full Network

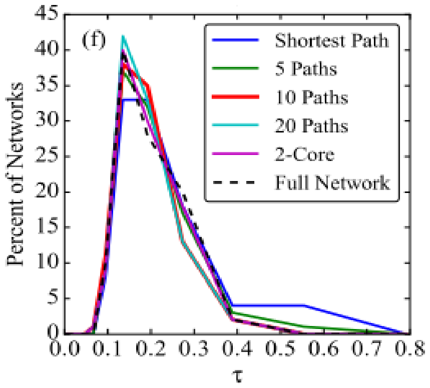


Subgraph corresponding to the network 10 shortest paths

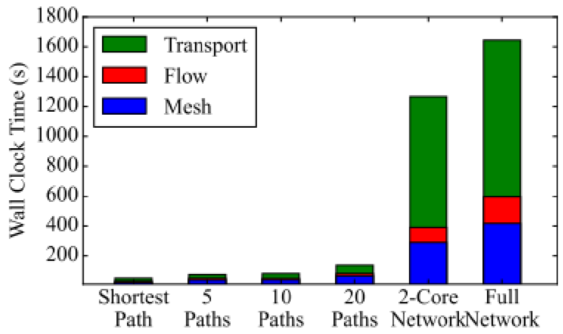


Reduced Network

[Hyman et al, 2017]



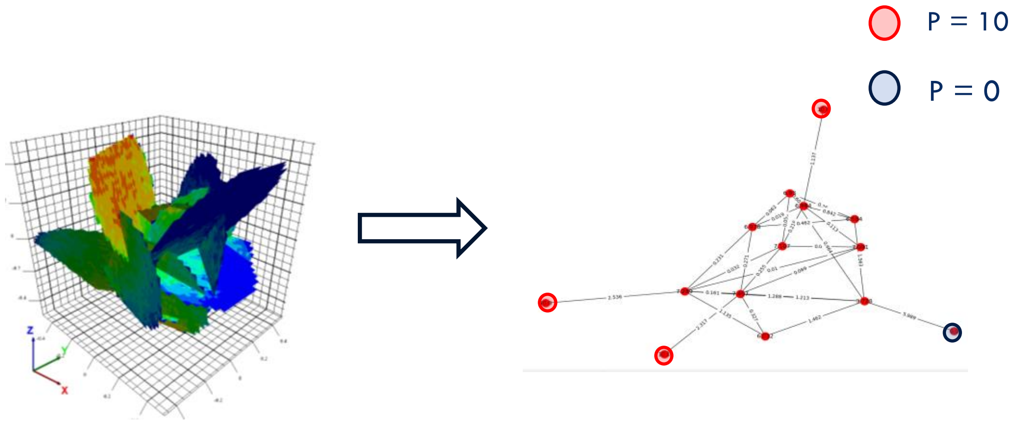
Distribution of first arrival times in different subnetworks.



Time required for meshing, flow and transport computation

Assessing DFN hydrological properties with Graphs

⇒ **Simulation tool for fast flow and transport computation**



No meshing.
Computation is directly made on nodes and edges.

- *Boundary conditions*
- *Mass conservation law of mass flow in node* i *:* $\sum_{j=1}^{N} Q_{ij} = 0$
- *Darcy's law :* $Q_{ij} = \boldsymbol{C_{ij}}\,(P_i - P_j)$

$\boldsymbol{C_{ij}}$: flow conductance between node i and node j
= graph edge attribute

⇒ Edges flow informations : flow Q_{ij}, velocities q_{ij}, travel time t_{ij}.

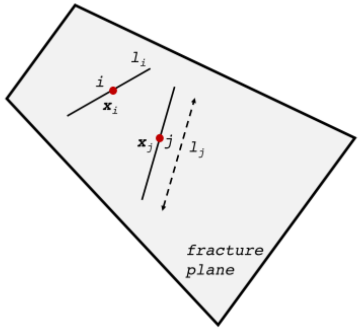
Assessing DFN hydrological properties with Graphs

⇒ **Simulation tool for fast flow computation**

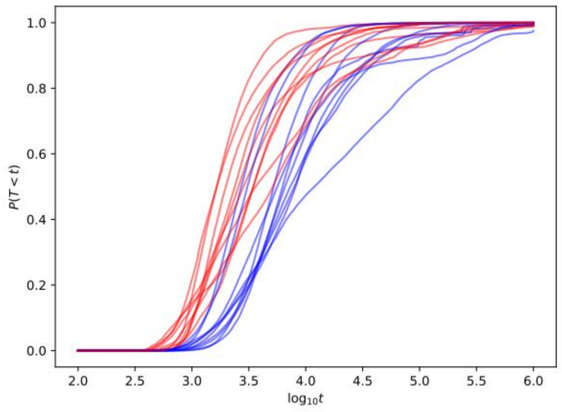
- Particle tracking in graphs

⇒ Nodes represent intersections
⇒ The conductance C_{ij} is defined on intersections geometrical informations.

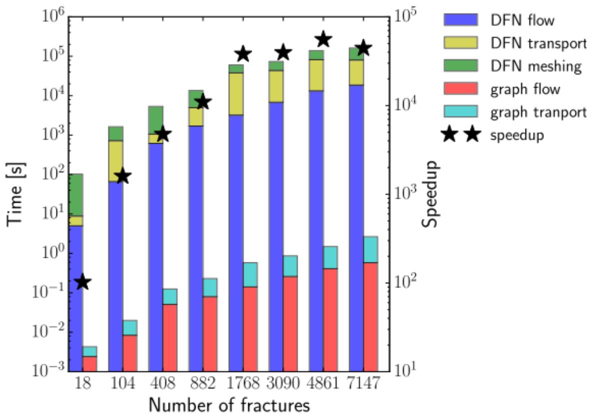
⇒ Breakthrough curves and CPU times



Geometrical information of fracture to map equivalent graph.



Breakthrough curves for DFN with 500 fractures (red) and equivalent graphs (blue).



CPU times for various steps.

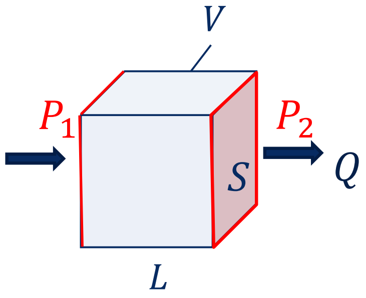
[Karra et al., 2017]

Assessing DFN hydrological properties with Graphs

⇒ **Simulation tool for fast flow computation**

- Equivalent permeability calculation

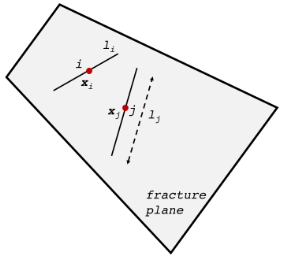
$$K_{eq} = \frac{Q.L}{S.(P_1 - P_2)}$$

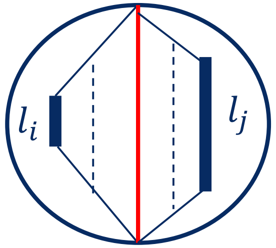


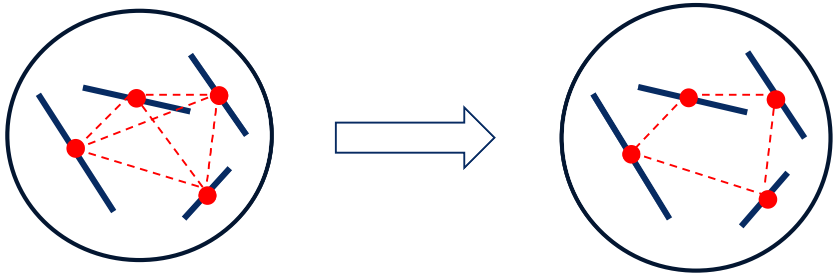
Importance of :

- Conductance C_{ij} definition, directly linked to flow Q intensity.

- Node and edge choice for graph.
  Nodes : Intersections, fractures, mixed ?
  Edges : remove crossing paths ?







Fracture Network sealing issues

First year of PhD : «*Sealing processes in Fracture Networks, models and hydrological consequences.*

⇒ *70% to 90% of fractures observed in SKB boreholes are sealed.*
⇒ Statistical graph-based approach.

Drill cores with sealed fractures (SKB).

- Tool for easily removing links with a given probability.

- Impact of the DFN topology to its 'robustness' to clogging, with graph indicators.
  Network robustness : ability of a network to continue performing in case of failures (Ellens et al., 2013).

- Clustering, Efficiency, Communities

- Betweenness : $$b_x = \sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{n_{ij}(x)}{n_{ij}}$$

Determines nodes (or edges) that occupy central positions in the Network

Conclusion

⇒ **Discrete Fracture Network (DFN) modeling :**

- essential
- complex
- computationally expensive

⇒ **Graph :**

- simple mathematical model
- edges / nodes
- extensively studied

⇒ **Graph Fracture Network (GFN)**

- characterize
- generate ?
- simplify
- simulate

Thanks for your attention !